JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 41, No. 5, September–October 2004

Precise Orbit Determination During Transfer Orbit Phase of GSAT-1

B. Subramanian,* Anatta Sonney,* N. V. Vighnesam,† and N. S. Gopinath‡
Indian Space Research Organization Satellite Centre, Bangalore 560 017, India

Orbit determination results are obtained for the GSAT-1 spacecraft launched by the Indian Space Research Organization using the first of its indigenously developed geosynchronous satellite launch vehicles. A description is given of the processes by which the mission operational ground-based orbit determination software estimated the satellite's achieved transfer orbit state. A comparison has been made between the determined transfer orbit elements and the injection parameters calculated immediately after launch by the inertial navigation system. A Monte Carlo simulation method that was used to estimate the errors in the orbit computed using the inertial navigation system data has been assessed. After injection, a major portion of the satellite's transfer orbit could not be tracked. To come to a definite conclusion of the achieved transfer orbit, investigations have been made using simulated tracking data and the actual tracking data of earlier, comparable, geostationary satellite missions. The orbit determination program used for the analyses is based on Cowell's formulation, the Gauss–Jackson numerical integration algorithm, and the batch weighted least-squares estimation technique. The performance of the tracking systems employed for the mission at Lake Cowichan, Canada; Fucino, Italy; Hassan, India; and Perth, Australia, has also been evaluated.

Nomenclature

A_S	=	effective cross-sectional area normal to the sun, km^2, m^2
A_V	=	spacecraft cross-sectional area projected normal to $\boldsymbol{V}_a$, km^2, m^2
a	=	semimajor axis, km, m
$\boldsymbol{a}_{\text{drag}}$	=	spacecraft acceleration caused by atmospheric drag, km/s^2, m/s^2
$\boldsymbol{a}_p$	=	vector sum of all of the perturbing accelerations, km/s^2, m/s^2
$\boldsymbol{a}_{\text{srp}}$	=	spacecraft acceleration caused by solar radiation pressure, km/s^2, m/s^2
C_D	=	aerodynamic drag coefficient
C_R	=	reflectivity of the surfaces of the satellite
c	=	speed of light, km/s
e	=	eccentricity
i	=	inclination, deg
k	=	number of measurements
M	=	mean anomaly, deg
m	=	spacecraft mass, kg
N	=	number of observations
P_{SR}	=	solar radiation pressure constant, N/m^2
r	=	$\lvert\boldsymbol{r}\rvert$, km, m
$\boldsymbol{r}$	=	Earth–satellite vector, km, m
$\boldsymbol{r}_{\text{ES}}$	=	Earth–sun vector, km, m
r_{SS}	=	$\lvert\boldsymbol{r}_{\text{SS}}\rvert$, km, m
$\boldsymbol{r}_{\text{SS}}$	=	sun–satellite vector, km, m
t_s	=	ranging signal travel time, ns
V_a	=	$\lvert\boldsymbol{V}_a\rvert$, km/s, m/s
$\boldsymbol{V}_a$	=	velocity vector of spacecraft relative to the rotating atmosphere, km/s, m/s
$\boldsymbol{v}$	=	velocity vector of spacecraft, km/s, m/s
W_j	=	weighting factor corresponding to each measurement residue
X, Y, Z	=	spacecraft position coordinates, km, m
X', Y', Z'	=	spacecraft velocity components in the coordinate directions, km/s, m/s
$\Delta\rho_j$	=	measurement residue, km, m, deg
μ	=	gravitational parameter, km^3/s^2, m^3/s^2
ρ	=	atmospheric density, kg/km^3, kg/m^3
ρ_{ran}	=	range, km
σ	=	standard deviation of the measurement, km, m, deg
υ	=	eclipse factor,
Ω	=	right ascension of ascending node, deg
ω	=	argument of perigee, deg
ω_E	=	rate of rotation of the Earth, rad/s

Introduction

THE orbit determination accuracy has been determined by various analyses that have been made earlier using high-fidelity orbit determination techniques. In one such method the accuracy has been determined using dense observations consisting of one observation per second for about 2 min.[1] It had been partially expected that the importance of the fit span with truly dense observational data would be diminished, but it was found that it was still important when highly precise orbits (less than a few tens of meters) are required. There are different methods like using a pulsed laser beam instead of radio wave for range measurement and using different force models for orbit determination, which can be employed to improve the accuracy of trajectory prediction and determination. Studies have been made to evaluate orbit determination accuracy based on difference in position approach.[2] Improved accuracy of the gravity and surface force models and improved performance of both the laser ranging and Doppler tracking systems in missions like TOPEX/POSEIDON have resulted in the achievement of rms errors within 4 cm in radial component of the spacecraft.[3] A high-precision software package developed by Jet Propulsion Laboratory (JPL) for geodetic surveying and for orbit determination of low Earth orbiters using global positioning system (GPS) uses a reduced dynamic technique[4] to produce the most accurate solution possible. A simulation interface allows this orbit determination system to be used in simulation analyses by which orbits and measurement data

Received 27 January 2003; revision received 5 September 2003; accepted for publication 8 September 2003.

*Engineer, Orbit Dynamics and Determination Section, Flight Dynamics Division, Airport Road, Vimanapura P.O.; subrab@isac.ernet.in; asonney@isac.ernet.in.

†Head, Orbit Dynamics and Determination Section, Flight Dynamics Division; vignes@isac.ernet.in.

‡Head, Flight Dynamics Division, Mission Development Group; nsgopi@isac.ernet.in.

are generated with a given set of models. A realistic orbit error is simulated by introducing appropriate model errors. A simulation technique incorporating six ground stations was applied to Geosat follow-on satellites and TOPEX/Poseidon satellites. The results of this study showed that when the number of ground tracking stations was increased to 13 from 6 there was a decrease in the radial orbit accuracy error. In a like manner, an orbit determination consistency study to discern an achieved geostationary transfer orbit (GTO) was made with simulated tracking data of two tracking stations, Lake Cowichan and Perth, and orbit solutions were estimated with long arc data (corresponding to about one transfer orbit period). The results were compared with orbit solutions obtained with actual tracking data and are reported in this paper.

An analysis of the operation of the orbit determination system used for the QuikSCAT mission gives the methods and results of processing measurement data. The operational orbit determination system has processed short segments of GPS pseudorange and carrier-phase data and obtained results that differ by less than 10 m from nominal orbit solutions.[5] The QuikSCAT orbit determination system produced rms position errors between 4 and 9 m for three-day arcs when pseudorange burst data solutions were compared with GPS solutions. It had been shown that the mission position accuracy requirements were achievable with GPS data but not with a backup orbit determination system using azimuth and elevation data. In a cognate manner, the operational orbit determination system that is currently being used by the Indian Space Research Organization (ISRO) to monitor the positions of all of its geostationary satellites was employed to carry out a study of the inertial navigation system (INS) results obtained from a launch vehicle. The first geosynchronous satellite launch vehicle (GSLV) of ISRO was GSLV-D1 that carried the GSLV Satellite-1 (GSAT-1) weighing 1540 kg. GSLV-D1 is also the first vehicle of ISRO that was used to demonstrate the application of cryogenic propulsion technology to generate the thrust required for realizing the injection velocity needed for placing a payload in geostationary transfer orbit. GSAT-1 was an experimental payload mainly intended to study the performance of this technology demonstrator. In particular, this paper highlights the performance of GSAT-1 orbit determination system during the transfer orbit stage of the GSAT-1 mission that was useful for the calibration of the accomplishment of GSLV-D1. The mission operational ground based-orbit determination system gave multiple solutions when tracking data of different periods and ground stations were used, and a determination was made of the actually achieved GTO.

Generally the range and angles should be zero calibrated before any operations because any imperfect part of the calibration causes biased tracking data. The fixed geometry of the satellite and tracking stations makes these biases difficult to be dealt with in orbit determination. There are techniques[6] for calibrating the biases in range and angle tracking by changing the satellite's longitude that leads to an improvement in orbit determination accuracy. The techniques show how the tracking biases can be estimated without having to depend on external calibration methods. For determining the GSAT-1 GTO, angle biases had to be estimated as unknown parameters in the orbit determination process especially in situations during the mission when only one station's tracking data were available. A description has been given here of the analyses that led to the conclusion of the solutions to the GSAT-1 GTO. A comparison has been made between the determined transfer orbit elements and the injection parameters calculated by the INS immediately after launch, and also a comparison is shown between the determined orbit results and the expected nominal orbit along with the launch vehicle's quoted dispersions. The coordinates of the ground stations involved in the initial phase of this mission are given in Table 1 and the performance of the ranging systems employed by these tracking stations has also been evaluated.

GSAT-1 Orbit Determination System

The orbits of all of the operational geostationary satellites of ISRO are determined by the orbit determination system developed at ISRO Satellite Centre, Bangalore, using a batch least-squares estimator to process tracking data from a network of stations configured for the mission. The orbit determination system (ODS) that consistently locates the position of a satellite during the GTO and the intermediate orbits during the geostationary Earth-orbiting (GEO) satellite missions of ISRO consists mainly of three programs: 1) Ephemeris Generator, 2) Tracking Data Preprocessor (TDPP), and 3) Orbit Determination Program (ODP).

Table 1 GSAT-1 tracking network

Station name	Longitude, °E	Latitude,°	Height, m
Hassan	76.096693	13.07044722	896.392
Perth	115.88767	−31.806592	23.5976
Lake Cowichan	235.92	48.83	336.9
Fucino	13.599498	41.976364	685.0715

Processing the Tracking Data

The International Telecommunication Satellite (INTELSAT) organization was engaged by ISRO for supporting the launch phase of the GSAT-1 mission. The two-way ranging method using a ranging signal that is radiated from the ground station of the tracking network directed towards the satellite was employed for tracking the satellite. GSAT-1 has two transponders to receive the signal and transmit it back to the ground station. The two transponders operate at uplink frequencies of 6419.32 and 6423.496 MHz and downlink frequencies of 4190.976 and 4194.0 MHz, respectively. The transponder delay varies from $1.205(10^{-5})$ s for a temperature of 248 K to $1.268(10^{-5})$ s corresponding to temperature of 328 K at the transponder's central frequency.

A burst-tone ranging (BTR) system was employed for generating the ranging signals. The burst-tone system provides a direct measurement of the round-trip delay to the spacecraft rather than the four phase measurements provided by a four-tone ranging system. The ranging system design includes the ability to perform zero-calibration measurements at the start of each ranging session to account for the propagation delay within the Earth station. The BTR system sends a short burst of a signal to the satellite. The received burst from the satellite becomes the delayed burst and includes the Earth station delay contribution. The zero range calibration allows the Earth station delay to be subtracted out. The same burst is also looped locally at the Earth station and is considered the reference burst. At the beginning of the ranging session, a zero range calibration is performed by a separate range measurement through a test-loop translator (TLT) located in the antenna hub to determine the Earth segment delay. Therefore, any changes in the Earth station delay are noted at the time of measurement. To maintain a high signal-to-noise ratio, the strength of the signal looped through the TLT is kept at a higher level than that received from the satellite. The basic principle is that the time interval between the reference and delayed tone bursts is used to compute the time delay to the satellite and hence the distance. For tracking geostationary satellites, the length of the burst tone is maintained short enough so that the range receiver is able to process the reference signal before the arrival of the delayed burst. The burst-tone length in this case is approximately 240 ms. The BTR system uses a preamble to provide a means to line-up the zero-crossings in the amplitude–time graph of the delayed signal with those of the reference signal. The burst tone is filtered to remove unwanted adjacent carriers, like telemetry, and to limit noise. Once the preambles of the reference and the delayed signals are lined up, each zero-crossing in the delayed signal can be matched to a zero-crossing in the reference signal. Each given zero-crossing in the reference tone is propagated around the Earth station's transmission and reception path, whereas the corresponding zero-crossing in the delayed signal has to propagate through the satellite. The zero-crossings are accurately time stamped, and hence the time difference between corresponding zero-crossings provides the round-trip delay (in nanoseconds) to the satellite without the Earth station delay.

The signal travel time t_s measured from the ground stations at Lake Cowichan, Perth, and Fucino were received at the mission control center at Hassan, India, from INTELSAT's controlling base at Washington, D.C. The round trip signal travel time is converted by TDPP into its equivalent range value $\rho_{\text{ran}} = ct_s(10^{-9})/2$, ($c = 299{,}792.458$ km/s), which is equal to the average of the uplink and downlink distance. The two-way range is obtained with an accuracy of 30 m (3σ). The two types of delays, namely, the spacecraft transponder's delay and the ground station delay, are subtracted from the raw range data before being used as input for orbit determination. Whereas the range data from external stations are received at the ISRO mission control center after being corrected for ground station delays, those from the Hassan tracking system are corrected for the ground station delay [$8.5925(10^{-5})$ s, approximately] by TDPP. The measurement of the direction of the maximum signal amplitude of the spacecraft gives the pointing angles in the topocentric system of the ground station. The aberration effects on the angle measurements caused by the relative motion between the incoming signal and the ground station are neglected. The sampling rate at which the measurements were obtained by the tracking system was one sample per minute, and each tracking session was of 20-min duration in the case of tracking data received from Lake Cowichan, Perth, and Fucino and three samples per minute and 10-min duration, respectively, in the case of the local tracking station, Hassan. The preprocessing program edits wild points, applies calibration corrections, converts the measurements' values given as output by the Telemetry Tracking and Command processor to engineering values, tags each measurement with appropriate time, and smoothes the data. The tracking data are also corrected for environmental refraction effects.

Precise Orbit Determination Technique

The orbit determination software uses the tracking data processed by TDPP as input for determining the converged orbit solutions. The perturbing forces that most affect the motion of the satellite are included in the differential equations of motion of the orbit model so that the predicted motion of the satellite will be as close as possible to the true motion of the satellite. The Gauss–Jackson predictor-corrector method is one of the widely used techniques in the orbit determination process. Studies of the theory upon which an eighth-order Gauss–Jackson predictor-corrector for numerical integration is based and the method by which the integrator is implemented in orbit propagation have been made recently.[7] The variable step size method and its effect on speed and accuracy has been discussed in detail. The results presented in this paper are the output of an orbit determination program using the Gauss–Jackson numerical integrator. The core of the orbit determination program is the model that generates the trajectory of the satellites. The orbit is generated by numerical integration of the acceleration equations.[8,9] The way in which the acceleration equations are set up is by Cowell's method, in which the equations of motion are expressed in terms of the total acceleration vector and solved directly for the position and velocity vectors. The equations of motion have been taken according to the Cowell's formulation:

$$\boldsymbol{r}'' + (\mu/r^3)\boldsymbol{r} = \boldsymbol{a}_p \tag{1}$$

Table 2 describes the mathematical models, the computational methods, and the perturbing accelerations that have been included in the governing differential equation to determine the actual acceleration of the satellite. Among the perturbations that have been considered in the trajectory generator, the methods by which those due to aerodynamic drag and solar radiation pressure have been incorporated in the orbit determination program are described next.

Atmospheric Drag

The acceleration caused by atmospheric drag can be expressed as follows:

$$\boldsymbol{a}_{\text{drag}} = -\left(\tfrac{1}{2}\right)\rho(C_D A_V/m)(V_a \boldsymbol{V}_a) \tag{2}$$

where

$$\boldsymbol{V}_a = \begin{bmatrix} X' + \omega_E Y \\ Y' - \omega_E X \\ Z' \end{bmatrix}$$

Table 2 Details of methods and models used in GSAT-1 orbit determination system

Description	Models/codes/constants
Newtonian potential of the Earth (central body perturbation)	EGM-96 geopotential model μ (for Earth) = 398,600.4415 km^3/s^2 Mean equatorial radius of Earth = 6,378.1363 km Earth's rotation rate = 7.29211585491836 (10^{-5}) rad/s Flattening coefficient = 1/298.257
Lunar and solar gravitational attraction (third-body perturbation)	JPL DE-403, ephemerides numerical model for computation of moon and sun coordinates
Drag caused by atmosphere in the transfer orbit stage of a GEO satellite mission	MSIS-90 for density calculation Area of cross section: 4 m^2 Spacecraft mass: 1,540 kg Coefficient of drag: 2.2
SRP coefficient estimation	Area of cross section: 4 m^2 Reflectivity: 1.25
Orbit generator	Cowell's formulation
Numerical integration (special perturbation technique)	Gauss–Jackson–Merson second sum method
Estimation technique	Batch weighted least-squares method
Sources of errors in orbit determination	Errors in spacecraft observations caused by tracking system instruments Uncertainties in tracking station coordinates Deficiencies in the physical and mathematical models employed for orbit determination

The quantity $(C_D A_V/m)$ is referred to as the ballistic coefficient and (A_V/m) is the area-to-mass ratio for drag calculation. Both of these quantities are important parameters in non-gravitational force modeling, especially for orbit determination of low-Earth-orbiting satellites. The variation in the area projected normal to the spacecraft's velocity and the errors associated with the models of the atmospheric density are the primary sources of uncertainty in the description of the drag force. In view of these uncertainties, estimation of ballistic coefficient in orbit determination is desirable for precise orbit determination. Because the orbit determination system that has been used for the analyses reported in this paper is meant for GEO satellite missions, the estimation of ballistic coefficient was not of primary significance and is not done. However, to determine the GSAT-1 transfer orbit solutions, the computation of deceleration of the satellite caused by drag is essential. In the GSAT-1 orbit determination program, the deceleration caused by aerodynamic drag is computed and accounted for in the orbit determination process only whenever the altitude of the satellite is below 2000 km.

Estimation of Solar Radiation Pressure Coefficient

Most of the GEO satellites have large solar panels, and considering the altitudes at which they will be orbiting the Earth during most of their lives the perturbation caused by solar radiation pressure (SRP) will have a more notable effect than that caused by aerodynamic drag. The effect of SRP on a GEO satellite is mainly to produce a long periodic cyclic perturbation in the orbit eccentricity while leaving the semimajor axis essentially unchanged. To improve the orbit determination accuracy, it becomes mandatory to estimate the consequences of SRP and the orbit determination system used for the GSAT-1 mission accounts for this effect. The acceleration of the satellite caused by solar radiation pressure is given by

$$\boldsymbol{a}_{\text{srp}} = (\upsilon P_{\text{SR}})(C_R A_S/m)(\boldsymbol{r}_{\text{SS}}/r_{\text{SS}}) \tag{3}$$

where $P_{\text{SR}} = 4.51(10^{-6})$ N/m^2, $\boldsymbol{r}_{\text{SS}} = \boldsymbol{r} - \boldsymbol{r}_{\text{ES}}$, $0 < C_{\text{R}} < 2$, and $0 < \upsilon < 1$. The ratio $(C_R A_S/m)$ is called the SRP coefficient, which changes continuously as the spacecraft travels in its orbit around the Earth. Because it is very difficult to physically obtain the exact values of both the effective cross-sectional area normal to the sun at every instant and the reflectivity C_R, the SRP coefficient is also numerically estimated as one of the unknown parameters along with the state vector. This process requires computation of partial derivatives of the measurements with respect to the modeled parameters. The SRP coefficient is determined as a function of the smoothed tracking observations by weighted least-squares estimation process. The partial derivatives of the current state with the epoch state (state transition matrix) are obtained numerically. In the estimation process of orbit determination, the numerical partial derivatives of measurements with respect to the SRP coefficient are augmented with the partial derivative matrix of six state parameters. In practice, the estimation of the SRP coefficient aided the determination of orbit solutions with very low final measurement rms values when the spacecraft was in drift orbit or in geostationary orbit, as tracking data of a longer duration (about 40 h and more) covering the full orbit will be available for the estimation process. In the case of a conventional geostationary transfer orbit (typically of 10-h orbit period), the maneuver schedules leave insufficient time for accurate SRP coefficient estimation.

Numerical Integration Method

For numerical integration, the equations of motion are reduced to first-order differential equations given next:

$$\boldsymbol{r}' = \boldsymbol{v} \tag{4}$$

$$\boldsymbol{v}' = (-\mu/r^3)\boldsymbol{r} + \boldsymbol{a}_p \tag{5}$$

Though the computation is slow with Cowell's method, it does not require that the magnitude of $\boldsymbol{a}_p$ be small. The second-order differential equations as just given are directly integrated numerically using the Gauss–Jackson integrator. In the numerical procedure the state vectors are obtained at discrete intervals of steps. The step size is altered from a minimum of 30 s, depending upon the size and shape of the orbit and the number of perturbations that are taken into consideration. A variable step method is used for the double integration. In the numerical integration algorithm the Gauss–Jackson predictor and corrector formulas[10] are used. Although the predictor-corrector method used in this algorithm requires a series of back values to be maintained, it nevertheless improves the efficiency and accuracy of the integration. The Gauss–Jackson method was chosen as the numerical integrator for the geostationary satellite orbit determination system because it is a more efficient method for circular orbits.

Estimation Techniques

Because the data system and dynamic models are both imperfect, no trajectory can be computed that fits the observations exactly. Thus the orbit determination system is designed to obtain a best estimate of the trajectory from the data in a statistical sense. The estimation techniques[11] can be divided into two broad categories, namely, 1) sequential estimation and 2) batch estimation. To obtain estimates of the state vector at the measurement times, a propagation of both the state vector and its covariance between the times of successive iterations is required, for which the sequential estimation or Kalman-filter algorithm can be used. The Kalman filter processes a single scalar or vector measurement at a time and yields sequential state estimates at the measurement times. For successfully using the basic Kalman filter, the deviations between the reference state and the estimated state must be small enough to neglect any nonlinearities in the system dynamics and the measurement modeling. In the case of sequential estimation, there is no need for storing measurements from previous time steps, which reduces the memory requirements of the computing system. On the other hand, batch estimation derives its name from the approach of accumulating a batch of data over some span of time and solving for the orbital state or parameters given an observation model, so as to minimize the squares of the difference between a computed and an observed trajectory. Because many data types are used in the observational data, weighting is assigned to each data set, and because a priori information is available about the uncertainty of the initial estimate it is also added to the estimation process. Because linearization takes place in relating the computed and observed values, the process is an iterative one, whereby differential corrections are applied to the state parameters until convergence to weighted least-squares solution is achieved. Because of the ease with which bad data can be edited with the weighted least-squares algorithm, and on account of its reliability in the presence of long time intervals between data sets, this algorithm was found to be the most suitable for the GEO satellite orbit determination system. In general the least-squares technique is more robust and easier to handle than a sequential estimation process.

Convergence and Rejection Criteria

The aim of estimation technique is to minimize the sum of the squares of the residuals, and therefore the sum of the squares of residuals is taken as the basis for convergence. At the end of every iteration, the sum of the squares of residuals, that is,

$$\sum_{j=1}^{k} W_j \Delta\rho_j^2$$

and the standard deviation σ are computed. The process is said to be converging if σ for the succeeding iteration is less than that of the previous one and deemed to be completed if the decrement is less than a user-defined tolerance value (typically 0.1 %)—when the process has reached a stage where the observations are not still being rejected. If σ for the succeeding iteration is more than that for the previous one and if it happens successively twice, then the diverging process is terminated. Bad data are weeded out in the following manner. At the end of every iteration, the overall standard deviation is evaluated. The weighted residual of every observation is compared with the overall standard deviation. If a particular observation has a weighted residual that is more than three times the overall standard deviation, then the corresponding observation is deleted from the observation file.

Analyses to Resolve the GSAT-1 GTO

Investigations were conducted on the following basis, and the ensuing results of these exercises are tabulated in this paper. First, depending upon the time and rate at which data were received from tracking stations, GSAT-1 transfer orbit was determined using tracking data of varying duration at the time immediate to the injection of the satellite. There seemed to be a difference in the perigee height determined by the ODP software when compared with the preliminary orbit determination (POD) results communicated from Sriharikota High Altitude Range (SHAR) to the Indian National Satellite (INSAT) Master Control Facility at Hassan, from where the geostationary satellites launched by ISRO are controlled. The possible reasons for this difference are discussed next.

Second, a study based on Monte Carlo estimation of errors in orbit results was conducted. The navigation team of ISRO inertial systems unit performed Monte Carlo simulations before the launch to estimate the errors in an orbit computed using INS data. The studies had shown 3σ estimates of perigee and apogee height errors to be 2.6 and 195 km, respectively. However, the orbit determination results based on ranging data showed that the INS orbit was in error by about $+2.4$ and -13 km in perigee and apogee heights, respectively. It was initially felt that such a combination of errors is not feasible, but based on further analysis of detailed Monte Carlo run results it was seen that a certain possible combination of INS errors can lead to the INS orbit inaccuracy observed in the mission.

Third, a study was made with the tracking data of two of the other ISRO GEO satellites, INSAT-2E and INSAT-3B, which are presently collocated at 83° east longitude, to assess orbit determination accuracy. The tracking scenario for this exercise was chosen to be identical with that of the GSAT-1 transfer orbit (T.O.) phase tracking conditions. GSAT-1 T.O. phase orbit determination accuracy was assessed with long arc (about one orbital period) simulated tracking data, and certain case studies that were considered to support orbit determination consistency are explained next.

GSAT-1 Transfer Orbit Determination

GSAT-1 was launched by GSLV-D1 from SHAR, India, and six orbital maneuvers were performed to achieve the targeted geostationary orbit. The GSAT-1 transfer orbit was determined to be (182.431 × 32,140.590) km by the ground-based ODS using 5 h 44 min of tracking data from Lake Cowichan and Perth. The injected orbit was found to be less by 561 m in perigee height and 3885.686 km in apogee height from the expected nominal orbit. The final orbit maneuver was performed on 23 April 2001, which resulted in an orbit of (33,806 × 35,665) km that fell short by about 1000 km in semimajor axis with respect to the expected geosynchronous orbit, for which the initial orbit into which the satellite was injected by the vehicle was one of the reasons. For the first time in ISRO's history of geostationary satellite missions, a perigee velocity augmentation technique was used to minimize the total velocity required for orbit raising to achieve the planned geostationary orbit. Because of the continuously changing nature of the satellite's orbit profile, the orbit determination process had to be repeated frequently after each of the maneuvers, using inputs available to the orbit determination software varying from very short arc data to long arc data, to realize the mission requirements.

The preliminary orbit determination of GSAT-1 was made at SHAR using the state vector data of the INS onboard the launch vehicle. According to the INS preliminary orbit results, the perigee height, apogee height, and inclination were 180.016 km, 32,153.531 km, and 19.284 deg, respectively. The GSAT-1 transfer orbit events related to orbit determination, such as satellite visibility from tracking stations, apsidal crossing times, and others are shown in Table 3. The epoch is given in UTC in year-month-day, hour (h)-minute (min)-second (s)-millisecond (ms), format throughout this paper. The satellite was injected into orbit at 10:30 UTC on 18 April 2001, and Lake Cowichan was the first station from which the satellite was visible from 11:19 UTC. But tracking data from Lake Cowichan was available only from 15:30 UTC because the telemetry down link had dropouts due to a ground configuration problem.

The orbit solutions obtained with 27 min of tracking data from Lake Cowichan are shown in Table 4. The orbit results showed perigee height, apogee height, and inclination of 171.98 km, 32,153.09 km, and 19.289 deg, respectively. Orbit determination was made with Lake Cowichan tracking data of 2 h 54 min duration, and the results obtained are shown in Table 4. The orbit results showed perigee height, apogee height, and inclination of 171.18 km, 32,154.74 km, and 19.252 deg, respectively. In both the short arc orbit determinations from single station tracking data, angle biases were not estimated and were not taken into consideration to fit the orbit.

To analyze the reasons for the difference of about 8 km in perigee heights, it was decided to obtain orbit solutions using tracking data of longer duration for comparison. On the day of launch, further orbit determination runs could not be made because of the preparations required for the first apogee motor firing (AMF), which was scheduled at the second apogee crossing of the satellite. Later, on 21 April 2001 with a total tracking data duration of 5 h 44 min the orbit was determined as shown in Table 4 (more details of this orbit determination are given later). The orbit results showed perigee height, apogee height, and inclination of 182.431 km, 32,140.590 km, and 19.280 deg, respectively. This was closer to the preliminary orbit results from INS.

The reasons for obtaining a closer perigee height to the INS results, from orbit determination results using 5 h 44 min of tracking data, can be attributed to the following factors:

The tracking data obtained for a period of 2 h 54 min from Lake Cowichan covered the trajectory arc after the apogee crossing as shown in Fig. 1. Because the tracking measurements near the very first perigee reached by GSAT-1 at the time of injection were not available, the orbit restitution around perigee was not accurate in the short arc transfer orbit determinations. Prior to the GSAT-1 mission, all of the GEO satellites of ISRO were launched from outside India, mostly by Arianespace from Kourou, French Guiana, and in these missions the tracking data corresponding to the transfer orbit

Table 3 GSAT-1 transfer orbit events in chronological order

Time (UTC) year-month-day h-min-s-ms	Event
2001-04-18	
10-30-16-783	GSAT-1 injection
11-19-00-532	Satellite rise at Lake Cowichan
15-08-00-947	Apogee 1 crossing
15-30-21-192	Start time of tracking data from Lake Cowichan
18-24-23-825	End time of tracking data from Lake Cowichan
19-30-35-484	Satellite set at Lake Cowichan
19-47-10-933	Perigee 2 crossing
20-11-44-244	Satellite rise at Perth
20-48-28-475	Start time of tracking data from Perth
22-06-58-967	End time of tracking data from Perth
2001-04-19	
00-26-19-941	Apogee 2 crossing
	Liquid apogee motor (LAM) firing 1 for the first maneuver
	LAM firing duration: 1177 s
	Total fuel spent for the maneuver: 165.6 kg

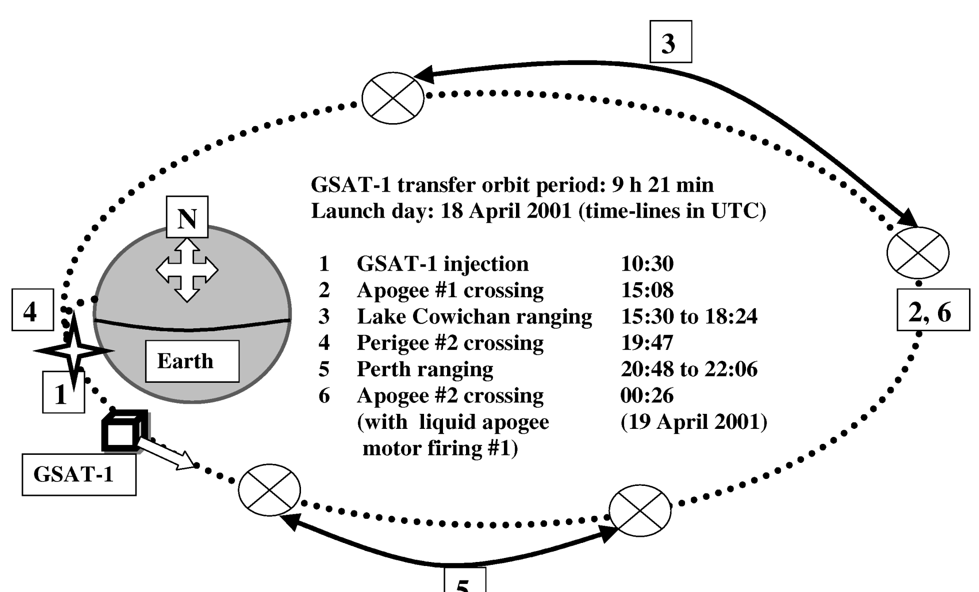

Fig. 1 GSAT-1 transfer orbit phase tracking scenario (not to scale).

Table 4 Comparison of short and long arc GSAT-1 T.O. determination solutions

Orbital elements at epoch (UTC): 2001-04-18 10-30-16-783, km, deg	Tracking duration for GSAT-1 T.O. determination: 27 min	2 h 54 min (without angle-bias estimation)	2 h 54 min (with angle-bias estimation)	5 h 44 min
Perigee height	171.98	171.181	182.773	182.431
Apogee height	32,153.09	32,154.744	32,139.8698	32,140.590
Inclination	19.2893607	19.2523656	19.2991652	19.2801093
ω	177.7923336	177.8273850	177.3935916	177.4151708
Ω	305.2213670	305.2077612	305.6037892	305.5774726
Mean anomaly	0.9132962	0.9279455	0.8833606	0.8912726

Table 5 Nominal orbit parameters for GSLV-D1/GSAT-1 mission

Launch window, UTC	Time from liftoff to injection: 1039.72 s Injection epoch, UTC: 2001-04-18, 10-30-19-720 — Injection parameters	GSAT-1 T.O. parameters
2001 April 18	Altitude: 203.733 km	a: 24,482.21147 km
Open: 10:13	Velocity: 10.23911 km/s	e: 0.73205
Close: 14:03	Flight-path angle: 86.962 deg	i: 19.27662 deg
	Velocity azimuth: 109.203 deg	ω: 178.01407 deg
Duration:	Latitude: −1.727°	Ω: 305.45614 deg
230 min	Longitude: 126.177°	M: 0.75941 deg

segment near perigee were available. The 5 h 44 min orbit determination solution was obtained using tracking data from two stations, namely, Lake Cowichan and Perth that covered the trajectory arc after the second perigee. Because the data were available from two stations, range data alone (angle data not used) were used for orbit determination. Also in the 27 min and 2 h 54 min short arc orbit determinations (Table 4), tracking data from only a single station (Lake Cowichan) were used. Therefore for the orbit solutions to converge, both range and angle data were used to fit the orbit and orbit determination runs were made with and without consideration of angle biases. The biases in angle measurements are generally caused by the tracking system instrumentation, and they are also a function of the longitudinal positions of both the spacecraft and the tracking Earth station. The orbit determination software can be used to give reasonable estimates of the tracking system angle biases if they are unavailable or unknown. The estimated angle biases will be closer to the true angle biases if the estimation process uses longer duration of tracking data that covers a full orbit or more. The change in the orbit solutions using the same set (2 h 54 min) of tracking data with estimated azimuth and elevation angle biases being considered for fitting the orbit is also shown in Table 4. These results show perigee height, apogee height, and inclination of 182.773 km, 32,139.869 km, and 19.299 deg, respectively, which are nearer to the INS preliminary orbit results compared to the orbit results obtained without using angle biases. Because the angle biases of the tracking systems employed at Lake Cowichan and Perth as a function of longitudes were not obtainable, and the duration for which tracking data were available for angle bias estimation was too short to rely upon at the time of determination of the transfer orbit, there was an uncertainty in concluding the definite transfer orbit results during the mission just before the first apogee maneuver.

Comparison Study

The nominal orbital parameters and INS orbit determination parameters are given in Tables 5 and 6, respectively. The segments of the GSAT-1 T.O. that were tracked by Lake Cowichan and Perth are shown in Fig. 1, and the tracking data corresponding to these segments of the orbit were used by the operational orbit determination system to obtain the 5 h 44 min transfer orbit determination solutions shown in Table 7. The corresponding measurement statistics are given in Table 8.

Nominal Orbit Comparison

The achieved GSAT-1 transfer orbit is compared with the expected nominal orbit in Table 9, which shows that while the

Table 6 INS injection parameters for GSLV-D1/GSAT-1 mission

Epoch (UTC): 2001-04-18, 10-30-16-783

State vector	Keplerian elements
X, km: −4,226.957098	a, km: 22,544.910291
Y, km: 5,038.868081	e: 0.709107
Z, km: −181.501702	i, deg: 19.284135
X', km/s: −7.629271	ω, deg: 177.615224
Y', km/s: −5.831150	Ω, deg: 305.468523
Z', km/s: −3.357873	M, deg: 0.863087
Altitude, km: 201.399512	Perigee altitude, km: 180.015568
Latitude,°: −1.580750	Apogee altitude, km: 32,153.531014
Longitude,°: 125.812303	Period, min: 561.477676
Velocity, km/s: 10.172679	Pitch, deg: 154.304137
Flight-path angle, deg: 87.023363	Yaw, deg: 6.826255
Azimuth inertial, deg : 109.221695	Roll, deg: 8.057219

Table 7 Definitive 5 h 44 min GSAT-1 T.O. phase precise orbit determination

Parameter description	Parameter value
Epoch: (UTC)	2001-04-18 10-30-16-783
Semimajor axis, km	22,539.6518320
Eccentricity	0.7089321
Inclination, deg	19.2801093
Argument of perigee, deg	177.4151708
Right ascension of ascending node, deg	305.5774726
Mean anomaly, deg	0.8912726
Tracking data from: (UTC)	2001-04-18 15-42-17-233
Tracking data to: (UTC)	2001-04-18 21-27-03-747

Table 8 Final rms values of measurements used for the 5 h 44 min GSAT-1 GTO determination

	Measurement statistics with final rms values					
	Range		Azimuth		Elevation	
Tracking station	N	rms, m	N	rms, deg	N	rms, deg
Perth	37	6.7	0	0.0	0	0.0
Lake Cowichan	243	8.0	0	0.0	0	0.0
Total	280	7.8	0	0.0	0	0.0

Table 9 Comparison of GSAT-1 T.O. phase realized orbit with nominal orbit parameters

Parameter, km, deg	Nominal	Determined (5 h 44 min OD)	Difference (OD – nominal)	Quoted dispersions (3σ)
Perigee height	181.870	182.431	0.561	5.0
Apogee height	36,026.276	32,140.590	−3, 885.686	675.0
Inclination	19.27662	19.280	0.00338	0.1
ω	178.014	177.415	−0.599	0.2

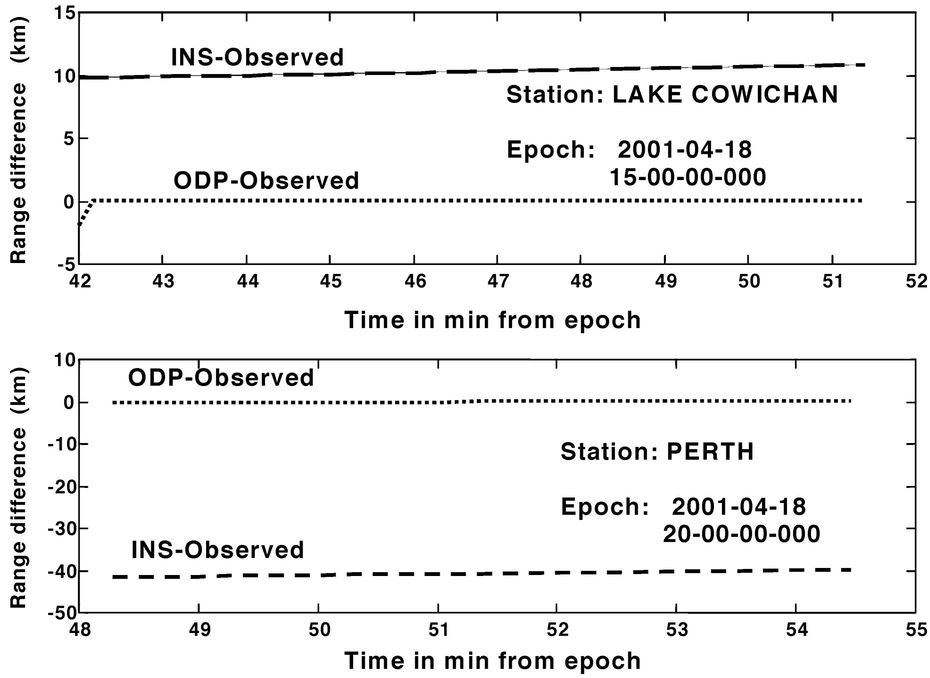

Fig. 2 GSAT-1: Comparison of range values.

nominal orbit is ($181.870 \times 36{,}026.276$) km the orbit achieved was ($182.431 \times 32{,}140.590$) km. The differences in perigee altitude, apogee altitude, and inclination were 0.561 km, 3,885.686 km, and 0.0034 deg, respectively, against the quoted dispersions (3σ) of 5 km, 675 km, and 0.1 deg, respectively. The large deviation in the apogee height is attributable to deficiency in the satellite's injection velocity.

INS Orbit Comparison

The 5 h 44 min transfer orbit determination results from the orbit determination software are compared in Table 10 with those from the INS preliminary orbit determination, which shows that the variations in perigee height, apogee height, and inclination are 2.415 km, 12.941 km, and 0.004 deg, respectively. Using the orbit results from 1) the orbit determination software and 2) INS orbit determination, orbit propagation was made to compute the range and angles measurements. The range measurements computed for Lake Cowichan and Perth tracking stations were compared with the actual tracking measurements, and these measurement residues were plotted as shown in Fig. 2. We found from this study that the range measurement residues calculated with ODP results with respect to the tracking data from Lake Cowichan and Perth are within 10 m as expected, whereas for the INS orbit determination results the range residues were about 10 and 40 km for Lake Cowichan and Perth stations, respectively, which clearly establishes that the actual measurements do not harmonize with the INS orbit determination results.

Study with INSAT-2E and INSAT-3B Missions' Data

An orbit determination study in a tracking scenario similar to that of GSAT-1 was conducted using the actual transfer orbit tracking data obtained during the initial phases of INSAT-2E and INSAT-3B missions, which took place in the years 1999 and 2000, respectively. The tracking data scenario is shown in Fig. 1. The INSAT–2E long arc T.O. phase orbit determination results and orbit determination results using INSAT-2E tracking data, obtained with a short arc similar to GSAT-1 tracking scenario, are given in Table 11. For the long arc (14 h 57 min) orbit determination both range and angle data of Hassan, Perth, and Lake Cowichan were used, with the total, final range measurement rms being 28 m and the total, final azimuth, and elevation rms being 0.0115 and 0.011 deg, respectively. For the short arc (3 h 50 min) orbit determination range and angle data of Hassan and Perth were used, with the total, final range measurement rms being 3.6 m and the total, final azimuth and elevation rms being 0.0131 and 0.0161 deg, respectively. These results show a variation of 159 and 52 m in perigee height and apogee height, respectively. The long arc and short arc (similar to GSAT-1 tracking scenario) INSAT-3B T.O. phase orbit determination results are shown in Table 11. Only the range data from Hassan, Perth, and Lake Cowichan were used for the long arc (15 h 44 min) orbit determination, and the total, final range measurement rms was 79 m. The range data of Hassan and Perth and angle data of Hassan were used to obtain the short arc (4 h 14 min) orbit determination solutions with the total, final range measurement rms being 10 m and the total, final azimuth and elevation rms being 0.0215 and 0.0139 deg, respectively. These results show a variation of 136 and 169 m in perigee and apogee height, respectively. The consolidated results from using INSAT-2E and INSAT-3B tracking data for GSAT-1 T.O. phase analysis are shown in Table 12. Because the orbit determination system used for these two missions was almost similar to that used for the GSAT-1 mission, it was possible to come to a conclusion that even if there had been no loss of tracking data as a result of a telemetry anomaly during the crucial first five hours after injection of the satellite, and had a longer arc orbit determination been possible during the transfer orbit phase of the GSAT-1 mission those orbit solutions would not have significantly differed from the 5 h 44 min orbit determination results.

Study with GSAT-1 Simulated Data

An orbit determination consistency study was conducted with simulated tracking data analogous to those from Lake Cowichan and Perth. The tracking data were generated in accordance with the operational environment of GSAT-1 using a priori state parameters. The simulated tracking data are ideal observations that are free from errors. The ephemeris orbital elements representing an ideal trajectory were also generated corresponding to the same apriori state parameters. Biases and noise values as shown in Table 13 were imposed on the measurements to corrupt them. The models used for generating the simulated tracking data were the same as those used during the mission. The orbit determination errors can be caused by the following aspects: measurement parameters such as biases and data noise; dynamic parameters such as gravitational constants of Earth, moon, and sun; nonspherical geopotential coefficients; accuracy of Earth station coordinates, and density models used for spacecraft aerodynamic drag estimation. All of these values were maintained identical for generating the simulated tracking data that was used for the orbit determination study. The converged state parameters obtained during the mission using the actual tracking data of 5 h 44 min duration were used as the initial guess. Orbit solutions were estimated using short arc (resembling the GSAT-1 T.O. orbit determination duration) and long arc (similar to about one GSAT-1 transfer orbit period of 9 h 21 min) simulated tracking data. These results are shown in Table 11. The estimated orbits have not shown any noticeable differences and are almost same: $(182.434 \times 32{,}140.503)$ km. The total, final range rms values were within 9 cm. The simulated tracking data helped to confirm that the extension to the usage of tracking data covering the full transfer orbit with a duration more than that which was available (5 h 44 min) during the actual orbit determination process at the time of the GSAT-1 mission would not have substantially altered the actually determined short arc orbit solutions.

Table 10 Comparison of GSAT-1 T.O. phase realized orbit with INS POD

Parameter, km, km/s, deg	Epoch (UTC): 2001-04-18 10-30-16-783 INS	ODP	Difference (ODP–INS)
a	22,544.910	22,539.652	−5.258
e	0.709107	0.7089321	−0.0001749
i	19.284135	19.2801093	−0.0040257
ω	177.615224	177.4151708	−0.2000532
Ω	305.468523	305.5774726	0.1089496
M	0.863087	0.8912726	0.0281856
X	−4,226.957098	−4,241.14129835	−14.1842
Y	5,038.868081	5,031.86191011	−7.006171
Z	−181.501702	−182.56065170	−1.0589497
X'	−7.629271	−7.622853423	0.006417577
Y'	−5.831150	−5.834209512	−0.003059512
Z'	−3.357873	−3.356121262	0.001751738
Perigee height	180.016	182.431	2.415
Apogee height	32,153.531	32,140.590	−12.941
Position difference	——	——	15.85557745
Velocity difference	——	——	0.007322191

Table 12 Consistency of orbit solutions obtained using tracking data of INSAT satellites with GSAT-1 T.O. scenario (comparison with respective long arc orbit determinations)

Satellite	Perigee height difference, m	Apogee height difference, m	Inclination difference, deg
INSAT-2E			
Long arc duration: 14 h 57 min	159	52	0.00379
Short arc duration: 3 h 50 min			
INSAT-3B			
Long arc duration: 15 h 44 min	136	169	0.00549
Short arc duration: 4 h 14 min			

Table 13 Errors superposed to generate simulated GSAT-1 tracking data

Measurement	Tracking system	Bias (1σ)	Noise (1σ)	Tracking station
Range	BTR	10 m	10 m	Lake Cowichan
Angles	BTR	0.0133 deg	0.0133 deg	Perth

Table 11 Orbital solutions obtained with other GEO spacecraft data, simulated tracking data, and different initial inputs (timelines in UTC)

	Tracking period			Orbit solutions, km, deg				
Satellite	Duration	From	To	Epoch	a, e	i, ω	Ω, M	Perigee height, Apogee height
INSAT-2E	14 h 57 min	1999-04-02 22-41-40-224	1999-04-03 13-39-10-132	1999-04-02 22-24-09-878	24,556.0337731, 0.7300328	3.9818018, 178.4962553	336.6654483, 2.0138295	251.189, 36,104.609
INSAT-2E	3 h 50 min	1999-04-03 03-23-30-119	1999-04-03 07-13-44-341	1999-04-02 22-24-09-878	24,556.0907253, 0.7300268	3.9855943, 178.3733886	336.7793228, 2.0160073	251.348, 36,104.557
INSAT-3B	15 h 44 min	2000-03-22 00-21-51-224	2000-03-22 16-06-05-984	2000-03-22 00-03-14-045	24,555.5146310, 0.7174602	6.9998070, 178.1472723	346.1634061, 10.1038400	559.775, 35,794.984
INSAT-3B	4 h 14 min	2000-03-22 05-20-30-121	2000-03-22 09-35-00-205	2000-03-22 00-03-14-045	24,555.5011071, 0.7174544	7.0053002, 178.0952841	346.2119151, 10.1032955	559.911, 35,794.815
	Simulated tracking data							
GSAT-1	6 h 4 min	2001-04-18 15-41-10-000	2001-04-18 21-46-00-000	2001-04-18 10-30-16-783	22,539.6060599, 0.7089314	19.2800897, 177.4151876	305.5774109, 0.8913140	182.434, 32,140.503
	Simulated tracking data							
GSAT-1	9 h 4 min	2001-04-18 15-41-10-000	2001-04-19 00-46-00-000	2001-04-18 10-30-16-783	22,539.6064136, 0.7089314	19.2800952, 177.4151987	305.5773871, 0.8913214	182.434, 32,140.503
				Solutions obtained with initial orbit input: (180.230 × 32140.590) km				
GSAT-1	5 h 44 min	2001-04-18 15-42-17-233	2001-04-18 21-27-03-747	2001-04-18 10-30-16-783	22,539.6416404, 0.7089314	19.2800453, 177.4144268	305.5783246, 0.8911560	182.442, 32,140.563
				Solutions obtained with initial orbit input: (180.016 × 32143.010) km				
GSAT-1	5 h 44 min	2001-04-18 15-42-17-233	2001-04-18 21-27-03-747	2001-04-18 10-30-16-783	22,539.6417176, 0.7089314	19.2800465, 177.4144288	305.5783206, 0.8911572	182.444, 32,140.563

Study with Perigee and Apogee Errors Ratio

A number of test cases to predict orbit solutions using Monte Carlo simulations were analyzed by the ISRO inertial systems unit (IISU) to study the expected ratio of perigee and apogee differences with respect to INS orbit results. In the case of the GSAT-1 mission, the 3σ prediction of apogee, perigee, and inclination errors were 195 km, 2.6 km, and 0.07 deg, respectively. But the apogee, perigee, and inclination differences between the mission's orbit determination system's results and those of INS were -12.941 km, 2.415 km, and -0.004 deg, respectively (as shown in Table 10). Whereas the apogee and inclination errors are well within the 3σ predictions, the perigee error is very close to the 3σ prediction of the Monte Carlo simulation. To analyze the reasons for this aspect of the predicted and observed errors, a study was made using the GSAT-1 orbit determination system. From statistical analyses conducted by the ISRO inertial systems unit using the data and achieved results from earlier satellite launches, a trend pertaining to the ratio between the differences in apogee and perigee heights was expected. Based on the expected trend, about 1/60 times the apogee difference of 12.941 km is added to the perigee altitude (180.016 km) calculated by the INS, to obtain a new perigee height: 180.23 km. Assuming the achieved apogee to be correct, the orbit determination results based on this $(180.23 \times 32{,}140.59)$ km orbit are given in Table 11. In this orbit determination process the initial range measurement rms for Perth range data (37 samples) was 7700 m, for Lake Cowichan (243 samples) 3200 m; and the overall (280 samples) initial range measurement rms was 4100 m. Hence from this study it was resolved that the injection parameters given by the GSLV-D1 inertial navigation system were in variation with the statistical deduction of the IISU results. The errors in the inertial navigation system's results can be caused by the following factors: 1) biases, nonlinearity, asymmetry, and misalignments of the accelerometer; 2) mass unbalance, misalignments, and asymmetry of the gyros; and 3) the initial alignment errors in the three cartesian coordinate directions X, Y, and Z. The ISRO inertial systems unit concluded that a combined effect of all of these sensor errors might have caused the observed orbital errors.

Test Case Study

The change in the value of the perigee height of a typical geostationary transfer orbit, whose eccentricity will be high, is sensitive more to a change in the eccentricity of the orbit rather than in its semimajor axis. Hence the eccentricity of a geostationary transfer orbit should be estimated to a high degree of accuracy. To verify the estimation of the eccentricity parameter of the GSAT-1 GTO, a study was made by determining the orbit based on an initial guess of a perigee altitude of 180.016 km calculated by INS, and the semimajor axis estimated by ODP namely 22,539.651832 km (Table 7), that is, for a $(180.016 \times 32{,}143.01)$ km orbit. A possible deviation of the estimated value of the semimajor axis from the true value was discounted, as the effect of even a small variation in the eccentricity on perigee height evaluation is greater than that caused by the former. Table 11 gives the GSAT-1 T.O. phase orbit determination results that are based on this initial orbit whose eccentricity is calculated to be 0.7090393 (compared to the eccentricity of 0.7089321 determined by the orbit determination software). In this orbit determination process the initial range measurement rms for Perth range data (37 samples) was 1900 m, for Lake Cowichan (243 samples) 2100 m; and the overall (280 samples) initial range measurement rms was 2100 m. This study showed that the INS-calculated perigee height of 180.016 km does not conform to the actual T.O. tracking measurements and hence is not appropriate.

Conclusions

In the study made to assess the GSAT-1 transfer orbit determination accuracy, the mission operational orbit determination software results were compared with the nominal and the inertial navigation system orbit determination results. The difference in apogee altitude calculated from the achieved orbit results and the nominal orbital parameters was about six times the launch vehicle quoted dispersion, which is attributable to the deficiency in injection velocity imparted to the spacecraft by the launch vehicle. In the comparison study between the mission operational orbit determination software solutions and the inertial navigation system orbit determination results, it was found that the computed range measurement residues with respect to measured range data from ground stations was much lower with orbit propagation using the former, whereas it was three orders more with orbit propagation using the latter. Hence it was inferred that orbit results determined using a batch least-squares method are more accurate than the orbital parameters derived from the injection parameters measured by the inertial navigation system of an experimental launch vehicle. There was a significant difference between the perigee height measured by the inertial navigation system and that which was determined at the time of the mission using tracking measurements corresponding only to the apogee segment of the orbit (because of contingencies faced during the mission). From the postmission analyses that were made using tracking data of longer duration, a reliable estimate of the perigee height was possible. The improved geometrical diversity of the measurements was the rationale behind this reliability. The observed differences in the apogee and perigee heights were a result of the combined effect of all of the inertial navigation system's sensor error parameters.

In this analysis, orbit determination runs were made to verify the effect of the usage of angle biases on orbit solutions. It was seen that there was a significant change in the orbit solutions that were obtained taking into consideration the angle biases estimated using tracking data of even a short duration of about 3 h, and these solutions were closer to the much more reliable orbit results uncorrupted by angle biases, obtained from a longer arc orbit determination. Also the parameters that can contribute to orbit determination errors were maintained identical to generate simulated tracking data analogous to those actually measured during the mission from the Earth stations. The orbit determination results obtained using the simulated tracking data covering short and long (all segments of one full transfer orbit) arcs agreed very closely with the actual transfer orbit results. Short arc and long arc orbit determinations using the actual transfer orbit tracking data corresponding to other geostationary satellites with duration similar to that which was obtained during the GSAT-1 transfer orbit stage showed little differences in apogee and perigee altitudes. From the transfer orbit determinations of other satellite missions using the same orbit determination system, it was possible to infer that the differences in apogee and perigee altitudes will not be considerable by an usage of an extended duration of tracking data beyond that which already covers most of the full orbit. With an orbit determination system that is employed for comparable satellite missions being almost unchanged, it is possible to use similar tracking scenarios and tracking data from the proven earlier missions to resolve an orbit determination issue facing constraints of a different mission.

Various analyses using tracking data that were obtained in an exigency during the satellite mission were planned and performed to confirm the actual true-of-date orbital elements of a geostationary satellite's transfer orbit. It was possible to verify the accuracy of the measured parameters of a launch vehicle's inertial navigational system using the mission operational ground-based orbit determination program by cross checking multiple sets of orbit solutions that were suspected to be the real solutions. To resolve the exactness of different sets of feasible, close orbit results, they were given as the initial guess for orbit determination using the actually measured tracking data. The computed initial measurement residues of the very first iteration of the orbit determination process gave indications about the accuracy of the initial orbit guess. The orbit solutions that matched the actual tracking data by giving minimal residues in the least-squares sense were decided to be the closest representation of the truly achieved orbit. It was eventually concluded that the ground-based mission operational orbit determination program using a batch least-squares estimation technique gave the closest solution to the precise transfer orbit achieved.

Acknowledgments

The authors thank P. S. Goel, Director; N. K. Malik, Deputy Director, Control and Missions Area; and P. J. Bhat, Group Director, Mission Development Group, ISRO Satellite Centre, Bangalore, for the continuous encouragement received from them to carry out analyses in the field of astrodynmics and orbit determination. The authors are also grateful to E. Vincent Zoby, Craig A. Kluever, and the anonymous referees for their meticulous, incisive and sagacious review of the manuscript and for providing substantial suggestions for its presentation.

References

[1]Vallado, D. A., and Carter, S. S., "Accurate Orbit Determination from Short-Arc Dense Observational Data," *Journal of the Astronautical Sciences*, Vol. 46, No. 2, 1998, pp. 195–213.

[2]Ogawa, M., Maeda, M., Sawabe, M., Hirota, M., and Yamamoto, Y., "Improvement of Orbit Determination and Prediction Accuracy of ADEOS/RIS," American Astronautical Society, AAS Paper 98-312, May 1998.

[3]Tapley, B. D., Ries, J. C., Davis, G. W., Eanes, R. J., Schutz, B. E., Shum, C. K., Watkins, M. M., Marshall, J. A., Nerem, R. S., Putney, B. H., Klosko, S. M., Luthcke, S. B., Pavlis, D., Williamson, R. G., and Zelensky, N. P., "Precise Orbit Determination for TOPEX/POSEIDON," *Journal of Geographical Research*, Vol. 99, No. C12, 1994, pp. 24,383–24,404.

[4]Irish, K., Gold, K., Born, G., Reichert, A., and Axelrad., P., "Precision Orbit Determination for the Geosat Follow-on Satellites," *Journal of Spacecraft and Rockets*, Vol. 35, No. 3, 1998, pp. 336–341.

[5]Thompson, B. F., Meek, M. C., Gold, K. L., Axelrad, P., Born, G. H., and Kubitschek, D. G., "Orbit Determination for the QuikSCAT Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 39, No. 6, 2002, pp. 852–858.

[6]Kawase, S., "Bias Estimation Technique for Geosynchronous Orbit Determination," American Astronautical Society, AAS Paper 99-184, Feb. 1999.

[7]Berry, M., and Healy, L., "Implementation of Gauss-Jackson Integration for Orbit Propagation," American Astronautical Society, AAS Paper 01-426, July–Aug. 2001.

[8]Chobotov, V. A., "Orbit Perturbations: Mathematical Foundations," *Orbital Mechanics*, 2nd ed., edited by V. A. Chobotov, AIAA Education Series, AIAA, Reston, VA, 1996, pp. 193–213.

[9]Cappellari, J. O., Velez, C. E., and Fuchs, A. J., "Mathematical Theory of the Goddard Trajectory Determination System," NASA TM-X-71106, April 1976.

[10]Vallado, D. A., "Special Perturbation Techniques," *Fundamentals of Astrodynamics and Applications*, 2nd ed., Microcosm Press, El Segundo, CA, and Kluwer Academic, Dordrecht, The Netherlands, 2001, pp. 489–563.

[11]Montenbruck, O., and Gill, E., "Orbit Determination and Parameter Estimation," *Satellite Orbits—Models, Methods, and Applications*, Springer-Verlag, Heidelberg, Germany, 2000, pp. 257–292.

C. A. Kluever
Associate Editor